

# Iron Ore Production and Logistics via Integer Programming

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## Long Term Planning

- Horizon is 10 years
- Years can be considered independently
- Demands can be aggregated to around 90
- Integral solution is not required
- A Linear Program is solved for each year
- Takes less than 2 min
- CPLEX 12 is used for all modules

## Medium Term Planning

- Horizon is from 1 to 3 years
- Granularity is month
- Stocks from one month to the next must be considered
- Local Mode: Runs for a subset of Productive Complexes
- Global Mode: Runs for all Supply Network
- Demands can be aggregated
- For one year plans: Local Mode averages 450 demands while Global Mode has 850
- For each demand there may be a minimum percentage of the demand that can be fulfilled
- No minimum percentage: Linear Programming
- Global mode with minimum percentage to fulfill: Runs in less than 5 min
- This is the most popular mode!

## Parameters

- $Cp(p)$  - Class of product  $p$ .
- $Ctp(p)$  - Category of product  $p$ .
- $Pre(i)$  - Instant preceding instant  $i$ .
- $Pos(i)$  - Instant succeeding instant  $i$ .
- $PP(cp)$  - Products from the class of products  $cp$ .
- $Loc(f)$  - Location of provider  $f$ .
- $Loc(pn)$  - Location of screening (peneiramento)  $pn$ .
- $Loc(c)$  - Location of client  $c$ .
- $Forn(l)$  - Provider of location  $l$ , should there be one.
- $Peneir(l)$  - Screening (Peneiramento) of location  $l$ , should there be one.
- $Cli(l)$  - Client of location  $l$ , stbo.
- $CIPr(d)$  - Demand Product  $d$ .
- $Cli(d)$  - Demand Client  $d$ .
- $MaxDem(d)$  - Maximum Fulfillment of demand  $d$  in the year.
- $MinDem(d)$  - Minimum Fulfillment of demand  $d$  in the year.
- $LoteMin(d)$  - Lot: Minimum Quantity in the Fulfillment of demand  $d$  at once
- $DemMensal(d, i)$  - Quantity of demand  $d$  fixed on instant  $i$ .
- $Pd(s, lo, ld, i, p)$  - Final product at destination of product  $p$  transportation using orig-dest  $lo$  to location  $ld$  using submodal  $s$  on instant  $i$
- $CapCS(l, s, i)$  - Load capacity (in tons) of submodal  $s$  at location  $l$  on instant  $i$
- $CapDS(l, s, i)$  - Unload capacity(in tons) of submodal  $s$  at location  $l$  on instant  $i$

## Parameters

- $CapEstoqueCp(l, cp, i)$  - Stock Capacity (in tons) on location  $l$  for product class  $cp$  on instant  $i$
- $CapEstoque(l, i)$  - Stock Capacity (in tons) on location  $l$  on instant  $i$
- $CapS(cp, s, i)$  - Capacidade do submodal  $s$  carregando produtos da classe  $cp$  no instante  $i$ .
- $CapVazao(lo, ld, s, i)$  - Throughput capacity from  $lo$  to  $ld$  for submodal  $s$  on instant  $i$ .
- $CapVazaoSr(sr, i)$  - Throughput capacity of sub-network  $sr$  on instant  $i$ .
- $CustoEstoque(l, i, cp, d)$  - Stock cost, during instant  $i$ , for one ton of product class  $cp$  in local  $l$  to fulfill demand  $d$ .
- $CustoForn(f, l, d, i, cp)$  - Custo de comprar do fornecedor  $f$  uma tonelada da classe de produto  $cp$  para atender a demanda  $d$  na localidade  $l$  no instante  $i$ .
- $CustoProd(l, i, cp)$  - Custo de produzir uma tonelada da classe de produto  $cp$  na localidade  $l$  no instante  $i$ .
- $CustoTransp(s, lo, ld, i, cp)$  - Custo de transportar uma tonelada da classe de produto  $cp$  no trecho de  $lo$  para  $ld$  utilizando o submodal  $s$  no instante  $i$ .
- $MinVazaoTr(lo, ld, s, i)$  - Mínimo de vazão (em número de submodais) no trecho de  $lo$  para  $ld$  para o submodal  $s$  no instante  $i$ .
- $MinVazaoTrCp(cp, lo, ld, s, i)$  - Mínimo de vazão (em número de submodais) para a classe de produto  $cp$  no trecho de  $lo$  para  $ld$  para o submodal  $s$  no instante  $i$ .
- $MinVazaoSr(sr, i)$  - Mínimo de vazão (em número de submodais) na subrede  $sr$  para o submodal  $s$  no instante  $i$ .
- $MinVazaoSrCp(cp, sr, i)$  - Mínimo de vazão (em número de submodais) para a classe de produto  $cp$  na subrede  $sr$  no instante  $i$ .
- $PercMaxGP(d, i, gp)$  - Percentual máximo de participação do grupo  $gp$  no atendimento da demanda  $d$  no instante  $i$ .
- $PercMinGP(d, i, gp)$  - Percentual mínimo de participação do grupo  $gp$  no atendimento da demanda  $d$  no instante  $i$ .

## Parameters

- $PercMaxCP(d, i, cp)$  - Percentual máximo de participação da classe  $cp$  no atendimento da demanda  $d$  no instante  $i$ .
- $PercMinCP(d, i, cp)$  - Percentual mínimo de participação da classe  $cp$  no atendimento da demanda  $d$  no instante  $i$ .
- $QtdMaxForn(f, i)$  - Quantidade máxima que pode ser comprada do fornecedor  $f$  no instante  $i$ .
- $QtdMaxForn(f, i, cp)$  - Quantidade máxima que pode ser comprada do fornecedor  $f$  da classe de produto  $cp$  no instante  $i$ .
- $QtdMinForn(f, i)$  - Quantidade mínima que pode ser comprada do fornecedor  $f$  no instante  $i$ .
- $QtdMinForn(f, i, cp)$  - Quantidade mínima que pode ser comprada do fornecedor  $f$  da classe de produto  $cp$  no instante  $i$ .
- $QtdProd(l, i)$  - Quantidade máxima produzida de qualquer produto na localidade  $l$  no instante  $i$ .
- $QtdProd(l, cp, i)$  - Quantidade máxima produzida do produto da classe  $cp$  na localidade  $l$  no instante  $i$ .
- $QtdMinProd(l, i)$  - Quantidade mínima que pode ser produzida na localidade  $l$  no instante  $i$ .
- $QtdMinProd(l, i, cp)$  - Quantidade mínima que pode ser produzida na localidade  $l$  da classe de produto  $cp$  no instante  $i$ .
- $QMin(d, pr, i)$  - Qualidade mínima da propriedade  $pr$  para a demanda  $d$  no instante  $i$ .
- $QMax(d, pr, i)$  - Qualidade máxima da propriedade  $pr$  para a demanda  $d$  no instante  $i$ .
- $QMeta(d, pr, i)$  - Qualidade meta da propriedade  $pr$  para a demanda  $d$  no instante  $i$ .
- $Qual(p, pr)$  - Qualidade da propriedade  $pr$  para o produto  $p$ .
- $Rec(d, i)$  - Receita por tonelada atendida da demanda  $d$  no instante  $i$ .
- $MinEstoqueCp(l, cp, i)$  - Estoque estratégico da classe de produto  $cp$  na localidade  $l$  no instante  $i$ .
- $MinEstoque(l, i)$  - Estoque estratégico total na localidade  $l$  no instante  $i$ .
- $EstIni(l, cp)$  - Estoque inicial da classe de produto  $cp$  na localidade  $l$ .

## Variables

- $x_{p,l,p,i}$  - quantity (in tons) produced of product  $p$  by location  $l$  on instant  $i$
- $x_{f,l,p,i}^f$  - quantity (in tons) of product  $p$  bought from provider  $f$  to be delivered at location  $l$  on instant  $i$
- $x_{s,lo,ld,i,p,pd}^t$  - quantity (in tons) of product  $p$  that will be converted into  $pd$  transported by submodal  $s$  from location  $lo$  to  $ld$  on instant  $i$ . Only orig-dest with handling factor will have  $p \neq pd$ .
- $x_{l,d,i,po,pd}^b$  - quantity (in tons) of product  $po$  blended to form product  $pd$  at location  $l$  on instant  $i$  to fulfill demand  $d$ .
- $e_{l,io,id,p}^l$  - quantity (in tons) of stock of product  $p$  at location  $l$  with closure on instant  $io$  and opening on instant  $id$
- $x_{c,d,l,p,i}^c$  - quantity received of product  $p$  by client  $c$  to fulfill demand  $d$  at location  $l$  on instant  $i$ .
- $x_{pen,l,p,i,pn,cf,pd}^e$  - quantity screened of product  $p$  at location  $l$  on instant  $i$  using screening configuration  $cf$  of screening  $pn$  resulting in product  $pd$
- $fmin_{f,i}^f$  - slack variable for constraint of minimum to be bought from provider  $f$  on instant  $i$
- $fmin_{cp,f,i}^f$  - slack variable for constraint of minimum to be bought of products of class  $cp$  from provider  $f$  on instant  $i$
- $fmin_{l,i}^p$  - slack variable for constraint of minimum to be produced at location  $l$  on instant  $i$
- $fmin_{cp,l,i}^p$  - slack variable for constraint of minimum to be produced of products of class  $cp$  at location  $l$  on instant  $i$
- $fmin_{l,i}^s$  - slack variable for constraint of minimum to be stocked at location  $l$  on instant  $i$ .
- $fmin_{cp,l,i}^s$  - slack variable for constraint of minimum to be stocked of products of class  $cp$  at location  $l$  on instant  $i$

## Variables

- $fvazmintr_{lo,ld,s,cp,i}$  - slack variable for constraint of minimum throughput of products of class  $cp$  for location  $lo$  to  $ld$  using submodal  $s$  on instant  $i$
- $fvazminsr_{sr,cp,i}$  - slack variable for constraint of minimum throughput of products of class  $cp$  for in the sub-network  $sr$  on instant  $i$
- $fmindem_{c,d,l}$  - slack variable for constraint of minimum to be fulfilled of demand  $d$  of client  $c$  at location  $l$ .
- $fqmin_{d,pr,i}$  - slack variable for constraint of minimum quality for property  $pr$  for demand  $d$  on instant  $i$
- $fqmax_{d,pr,i}$  - slack variable for constraint of maximum quality for property  $pr$  for demand  $d$  on instant  $i$
- $fqmetaU_{d,pr,i}$  - slack variable for constraint of target (upper) quality for property  $pr$  for demand  $d$  on instant  $i$
- $fqmetaL_{d,pr,i}$  - slack variable for constraint of target (lower) quality for property  $pr$  for demand  $d$  on instant  $i$
- $xbldm_{d,i}$  - [MT] binary variable identifying whether demand  $d$ , with minimum lot, will have a (partial) fulfillment or not on instant  $i$ .
- $fifixdem_{d,i}$  - [MT] slack variable for partial fulfillment of demand  $d$  fixed on instant  $i$
- $fmdistunifano_{d,i}$  - [MT] slack variable (lower) for annual distribution of demand  $d$  on instant  $i$
- $fpdistunifano_{d,i}$  - [MT] slack variable (upper) for annual distribution of demand  $d$  on instant  $i$
- $fmdistunif_{d,i}$  - [MT] slack variable (lower) for the distribution of demand  $d$  on instant  $i$
- $fpdistunif_{d,i}$  - [MT] slack variable (upper) for the distribution of demand  $d$  on instant  $i$

## Objective Function

$$\begin{aligned}
\text{MAX} \quad & \sum_{p \in P} \sum_{c \in C} \sum_{d \in D} \sum_{i \in I} \sum_{l \in L} \text{Rec}(d, i) \times xc_{c,d,l,p,i} \\
& - \sum_{p \in P} \sum_{i \in I} \sum_{l \in L} \text{CustoProd}(l, i, Cp(p)) \times xpl_{p,i} \\
& - \sum_{p \in P} \sum_{f \in F} \sum_{i \in I} \sum_{l \in L} \text{CustoForn}(f, l, i, Cp(p)) \times xfl_{p,i} \\
& - \sum_{p \in P} \sum_{s \in S} \sum_{d \in D} \sum_{i \in I} \sum_{lo \in L} \sum_{ld \in L} \text{CustoTransp}(s, lo, ld, i, Cp(p)) \times xt_{s,lo,ld,d,i,p,Pd(s,lo,ld,i,p)} \\
& - \sum_{p \in P} \sum_{io \in I} \sum_{id \in I} \sum_{l \in L} \text{CustoEstoque}(l, io, Cp(p), d) \times el_{l,io,id,p}
\end{aligned}$$

## Product Flow Conservation

$$\begin{aligned}
& el_{l,Pre(i),i,p} + xpl_{p,i} + xfl_{p,i} + \sum_{lo \in L} \sum_{s \in S} \sum_{po \in P} xt_{s,lo,l,i,po,p} \\
& + \sum_{d \in D} \sum_{pd \in P} xbl_{d,i,p,pd} + \sum_{cf \in CF(pn)} \sum_{po \in P} xpen_{l,po,i,pn,cf,p} \\
& = \\
& \sum_{d \in D} \sum_{po \in P} xbl_{d,i,po,p} + \sum_{ld \in L} \sum_{s \in S} xt_{s,l,ld,i,p,Pd(s,l,ld,i,p)} \\
& + \sum_{cf \in CF(pn)} \sum_{pd \in P} xpen_{l,p,i,pn,cf,pd} + el_{l,i,Pos(i),p} + \sum_{d \in D} xc_{c,d,l,p,i}
\end{aligned}$$

$$\forall p \in P \quad \forall i \in I \quad \forall l \in L \quad [f = \text{Forn}(l), pn = \text{Peneir}(l), c = \text{Cli}(l)]$$

**Load**

$$\sum_{ld \in L} \sum_{p \in P} \frac{x_{s,l,ld,i,p}^{t, Pd(s,l,ld,i,p)}}{CapS(Cp(p), s, i)} \leq CapCS(l, s, i) \quad \forall l \in L \quad \forall s \in S \quad \forall i \in I$$

**Unload**

$$\sum_{lo \in L} \sum_{p \in P} \frac{x_{s,lo,i,p}^{t, Pd(s,lo,i,p)}}{CapS(Cp(p), s, i)} \leq CapDS(l, s, i) \quad \forall l \in L \quad \forall s \in S \quad \forall i \in I$$

**Class of Product**

$$\sum_{p \in P(cp)} e_{l,i,Pos(i),p} \leq CapEstoqueCp(l, cp, i) \quad \forall cp \in CP \quad \forall i \in I \quad \forall l \in L$$

**Whole Stock**

$$\sum_{p \in P} e_{l,i,Pos(i),p} \leq CapEstoque(l, i) \quad \forall i \in I \quad \forall l \in L$$

**Maximum Fulfillment**

$$\sum_{p \in PD(d)} \sum_{i \in I} x_{c,d,l,p,i} \leq MaxDem(d) \quad \forall d \in D \quad [c = Cli(d), l = Loc(c)]$$

**Minimum Fulfillment**

$$\sum_{p \in PD(d)} \sum_{i \in I} x_{c,d,l,p,i} + f_{mindem_{c,d,l}} \geq MinDem(d) \quad \forall d \in D \quad [c = Cli(d), l = Loc(c)]$$

**Production by Class of Product and Instant**

$$\sum_{p \in PP(cp)} x_{p,l,p,i} \leq QtdProd(l, cp, i) \quad \forall i \in I \quad \forall cp \in CP \quad \forall l \in L$$

**Production by Instant**

$$\sum_{p \in P} x_{p,l,p,i} \leq QtdProd(l, i) \quad \forall i \in I \quad \forall l \in L$$

**Transport Capacity Orig-Dest and Instant**

$$\sum_{p \in P} \frac{xt_{s, lo, ld, i, p, Pd(s, lo, ld, i, p)}}{CapS(Cp(p), s, i)} \leq CapVazao(lo, ld, s, i) \quad \forall i \in I \quad \forall s \in S \quad \forall lo \in L \quad \forall ld \in L$$

**Minimum Throughput Orig-Dest, Class of Product and Instant**

$$\sum_{p \in PP(cp)} \frac{xt_{s, lo, ld, i, p, Pd(s, lo, ld, i, p)}}{CapS(cp, s, i)} + fvazmintr_{lo, ld, s, cp, i} \geq MinVazaoTrCp(cp, lo, ld, s, i)$$

$$\forall i \in I \quad \forall cp \in CP \quad \forall s \in S \quad \forall lo \in L \quad \forall ld \in L$$

**Minimum Throughput Orig-Dest and Instant**

$$\sum_{p \in P} \frac{xt_{s, lo, ld, i, p, Pd(s, lo, ld, i, p)}}{CapS(Cp(p), s, i)} \geq MinVazaoTr(lo, ld, s, i) \quad \forall i \in I \quad \forall s \in S \quad \forall lo \in L \quad \forall ld \in L$$

**Transport Capacity by Sub-Network and Instant**

$$\sum_{s \in S} \sum_{lo \in L} \sum_{ld \in L} \sum_{p \in P} \frac{x_{s,lo,ld,i,p,Pd(s,lo,ld,i,p)}}{CapS(Cp(p), s, i)} \leq CapVazaoSr(sr, i) \quad \forall sr \in SR \quad \forall i \in I$$

**Minimum Throughput by Sub-Network, Class of Product and Instant**

$$\sum_{s \in S} \sum_{lo \in L} \sum_{ld \in L} \sum_{p \in PP(cp)} \frac{x_{s,lo,ld,i,p,Pd(s,lo,ld,i,p)}}{CapS(cp, s, i)} + fvazmins_{sr,cp,i} \geq MinVazaoSrCp(cp, sr, i)$$

$$\forall i \in I \quad \forall cp \in CP \quad \forall sr \in SR$$

**Minimum Throughput by Sub-Network and Instant**

$$\sum_{s \in S} \sum_{lo \in L} \sum_{ld \in L} \sum_{p \in P} \frac{x_{s,lo,ld,i,p,Pd(s,lo,ld,i,p)}}{CapS(Cp(p), s, i)} \geq MinVazaoSr(sr, i) \quad \forall i \in I \quad \forall sr \in SR$$

**Demand Minimum Quality**

$$\sum_{lb \in LB} \sum_{p \in P} \sum_{po \in P \setminus PC(p)} Qual(po, pr) \times xb_{lb,d,i,po,p} + fqmin_{d,pr,i} \geq$$

$$QMin(d,pr,i) \times \sum_{lb \in LB} \sum_{p \in P} \sum_{po \in P \setminus PC(p)} xb_{lb,d,i,po,p}$$

**Demand Maximum Quality**

$$\sum_{lb \in LB} \sum_{p \in P} \sum_{po \in P \setminus PC(p)} Qual(po, pr) \times xb_{lb,d,i,po,p} - fqmax_{d,pr,i} \leq$$

$$QMax(d,pr,i) \times \sum_{lb \in LB} \sum_{p \in P} \sum_{po \in P \setminus PC(p)} xb_{lb,d,i,po,p}$$

**Demand Target Quality**

$$\sum_{lb \in LB} \sum_{p \in P} \sum_{po \in P \setminus PC(p)} Qual(po, pr) \times xb_{lb,d,i,po,p} + fqmetaU_{d,pr,i} - fqmetaL_{d,pr,i} =$$

$$QMeta(d,pr,i) \times \sum_{lb \in LB} \sum_{p \in P} \sum_{po \in P \setminus PC(p)} xb_{lb,d,i,po,p}$$

$$\forall i \in I \quad \forall pr \in PR \quad \forall d \in D$$

**Demand - Maximum Percentage of a Group of Products in the Blending**

$$\sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in PG(gp) \setminus PC(p)} x_{lb,d,i,po,p} \leq PercMaxGP(d, i, gp) \times \sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in P \setminus PC(p)} x_{lb,d,i,po,p}$$

$$\forall i \in I \quad \forall gp \in GP \quad \forall d \in D$$

**Demand - Minimum Percentage of a Group of Products in the Blending**

$$\sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in PG(gp) \setminus PC(p)} x_{lb,d,i,po,p} \geq PercMinGP(d, i, gp) \times \sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in P \setminus PC(p)} x_{lb,d,i,po,p}$$

$$\forall i \in I \quad \forall gp \in GP \quad \forall d \in D$$

**Demand - Maximum Percentage of a Product in the Blending**

$$\sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in PP(cp) \setminus PC(p)} x_{b_{lb,d,i,po,p}} \leq PercMaxCP(d, i, cp) \times \sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in P \setminus PC(p)} x_{b_{lb,d,i,po,p}}$$

$$\forall i \in I \quad \forall cp \in CP \quad \forall d \in D$$

**Demand - Minimum Percentage of a Product in the Blending**

$$\sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in PP(cp) \setminus PC(p)} x_{b_{lb,d,i,po,p}} \geq PercMinCP(d, i, cp) \times \sum_{lb \in LB} \sum_{p \in PD(d)} \sum_{po \in P \setminus PC(p)} x_{b_{lb,d,i,po,p}}$$

$$\forall i \in I \quad \forall cp \in CP \quad \forall d \in D$$

**Maximum buy of iron ore from a provider and class of product**

$$\sum_{p \in PP(cp)} x_{f,l,p,i} \leq QtdMaxForn(f, i, cp) \quad \forall f \in F \quad \forall i \in I \quad \forall cp \in CP \quad [l = Loc(f)]$$

**Maximum buy of iron ore from a provider**

$$\sum_{p \in P} x_{f,Loc(f),p,i} \leq QtdMaxForn(f, i) \quad \forall f \in F \quad \forall i \in I$$

**Minimum buy of iron ore from a provider and class of product**

$$\sum_{p \in PP(cp)} x_{f,Loc(f),p,i} + fminf_{cp,f,i} \geq QtdMinForn(f, i, cp) \quad \forall f \in F \quad \forall i \in I \quad \forall cp \in CP$$

**Minimum buy of iron ore from a provider**

$$\sum_{p \in P} x_{f,Loc(f),p,i} + fminf_{f,i} \geq QtdMinForn(f, i) \quad \forall f \in F \quad \forall i \in I$$

**Minimum Production Per location and class of product**

$$\sum_{p \in PP(cp)} x_{p,l,p,i} + f_{minpcp_{cp,l,i}} \geq QtdMinProd(l, i, cp) \quad \forall l \in L \quad \forall i \in I \quad \forall cp \in CP$$

**Minimum Production by Location**

$$\sum_{p \in P} x_{p,l,p,i} + f_{minp_{l,i}} \geq QtdMinProd(l, i) \quad \forall l \in L \quad \forall i \in I$$

**Minimum Stock by Location and class of product**

$$\sum_{p \in P(cp)} e_{l,i,Pos(i),p} + f_{minelcp_{cp,l,i}} \geq MinEstoqueCp(l, cp, i) \quad \forall cp \in CP \quad \forall i \in I \quad \forall l \in L$$

**Minimum Stock by Location**

$$\sum_{p \in P} e_{l,i,Pos(i),p} + f_{minel_{l,i}} \geq MinEstoque(l, i) \quad \forall i \in I \quad \forall l \in L \quad (1)$$

**Uniform Distribution of the Slacks**

$$fpdistunifano_{d,i} + fmdistunifano_{d,i} = \frac{1}{12} \sum_{i' \in I} (fpdistunifano_{d,i'} + fmdistunifano_{d,i'}) + fpdistunif_{d,i} - fmdistunif_{d,i}$$

$$\forall i \in I \quad \forall d \in D$$

**Percentage of Products in the screening**

$$xpen_{l,p,i,pn,cf,pd} = PercPen(pn, cf, pd) \sum_{pd' \in PP(cf)} xpen_{l,p,i,pn,cf,pd'}$$

$$\forall pn \in PN \quad \forall cf \in CF(pn) \quad \forall pd \in PP(cf) \quad \forall i \in I \quad [p = Prod(cf), l = Loc(pn)]$$

**Minimum Quantity (Lot) for each demand on instant i**

$$x_{c,d,l,p,i} \geq \text{LoteMin}(d) \times x_{blm_{d,i}} \quad \forall i \in I \quad \forall d \in D \quad \forall p \in P \quad [c = Cli(d), l = Loc(c)]$$

**Maximum delivery for each demand on instant i**

$$x_{c,d,l,p,i} \leq \text{MaxDem}(d) \times x_{blm_{d,i}} \quad \forall i \in I \quad \forall d \in D \quad \forall p \in P \quad [c = Cli(d), l = Loc(c)]$$

**Fixed Demand Fulfillment on instant i**

$$x_{c,d,l,p,i} = \text{DemMensal}(d, i) - \text{fmxixdem}_{d,i} \quad \forall i \in I \quad \forall d \in D \quad \forall p \in P \quad [c = Cli(d), l = Loc(c)]$$

**Uniformly Distributed Demand on instant i**

$$x_{c,d,l,p,i} = \text{DemUnif}(d, i) + \text{fpdistunifano}_{d,i} - \text{fmdistunifano}_{d,i}$$

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## Short Term Planning

- Horizon is from 3 to 5 months
- Granularity is day
- Runs for each productive system
- Shall run for a higher hierarchy (complexes seen as mines)
- Shipments CANNOT be aggregated
- Number of iron ore “sale” demands considered is around 30 (aggregated)
- Number “feed” demands (for pellets) considered is around 50 (aggregated)
- Number of shipments is about 150
- Number of stock piles go above 200
- The MIP model has about 200.000 binary or integer variables
- Continuous variables count is over 300.000

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This order is by product. Hard to impose this order in the MIP model, since some shipments may not be attended at all.
- Shipments must be totally completed before the next one starts.

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### 11 Repeat steps 6 to 10 until all the shipments were tried out

## Mutation

- 1 Selects variable types to be fixed
- 2 Removing all CPLEX heuristics, run as MIP a very short time and return the Best Node.
- 3 Randomly select 40% of the variables of the type chosen to the value in the current solution.
- 4 Run the complete model
- 5 Repeat steps 2 to 4 until the gap between the current objective function value and the Best Node on step 2 becomes smaller enough (1% is used)

## Try to fulfill shipments

- Fix all completely fulfilled shipments
- For all other shipments (including the partially fulfilled)
  - 1 Set all as not allowed to be fulfilled
  - 2 One by one, allow all possible blending and and all possible transportation. If any of those variables were fixed, unfix it.
  - 3 Run the complete model.
  - 4 Go sliding one day at a time the D value until the last day in the planning horizon is reached. Stop if the shipment considered is completely fulfilled (we try for 10 days, then feasibility is checked on the last day of the horizon).
  - 5 If the shipment could be fulfilled, fix it as fulfilled. Otherwise, fix it as not to be fulfilled and do not try to fulfill any other shipment for the same product.
  - 6 Fix back to zero all the blending and transportation variables that were fixed on step 33 of the main algorithm and were not used to fulfill this shipment.
  - 7 Repeat steps 2 to 10 until until all unfulfilled shipments were tried out.

## Allow the next set of priority shipments to be fulfilled

- 1 Add a constraint that forbids completing the fulfillment of the current set of priority shipments before the prior shipments are completely fulfilled
- 2 For all the shipments now considered, set  $D$  to the first day after the last shipment of the same product was fulfilled.

## Considerations

- Even if we had a method capable of optimally solving the Short Term problem for some time the solution generate would not be considered acceptable by users
- This is due to factors such as the incomplete description of the operational constraints, the distance of the decision maker from the stake holder or historical reasons
- Nevertheless, this Short Term Planning problem is just a less studied combination of well known Optimization Problems: A Multiple Discretized Diet Problem over Space and Time with Time Windows
- The above presented approach is the result of an already 6 month old tuning process with a daily contact with the decision maker. This solver development is more than one year old
- No computational experience beyond decision maker analysis yet
- The time allowed to provide a plan should not exceed 30 min
- This is a global problem with an enormous economic impact! Also social, ecological...

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- The MDDSTTWP have a number of characteristics we like on problems.
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- **And it is green!**

Thank You!